

THE SAGE ENGINE

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1. SAGE

Sage is an open source alternative to Maple, Mathematica, and Matlab. The program provides all of the standard features expected from such a system: arbitrary precision arithmetic, symbolic integration and differentiation, two-dimensional plotting of functions, matrix algebra, and much more. It can be downloaded from <https://www.sagemath.org>. That site will in turn direct macOS users to a specific site with two install packages for Sage, one for Arm machines and one for Intel machines. We strongly recommend the latest version, currently SageMath 9.5. Sage is updated regularly, so by the time you read this document there may be a later version available.

The Sage developers have dramatically improved installation for the Macintosh, so the many installation steps we used to provide in this document can be replaced with the words “follow their instructions.”

To use Sage, drag the `sage.engine` file in this folder to \sim /*Library/TeXShop/Engines*, the folder of active engines for TeXShop. This step need only be done once, when you first install Sage. This engine will work with Sage 9.4 and higher.

2. HOW SAGETEX WORKS

This document ends with sample source code and output illustrating how `sagetex` works. In the source file, the initial line

```
% !TEX TS-program = sage
```

tells TeXShop to process the file using the sage engine; this engine first calls `pdflatex`, then calls `sage`, and finally calls `pdflatex` again. The remaining lines in the preamble are standard LaTeX commands, except the required line

```
\usepackage{sagetex}
```

In the remaining source, `sage` commands are entered within lines of the form

```
\sage{...}
```

These lines cause `sage` to process commands and output LaTeX source fragments, which become part of the LaTeX document.

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Notice in particular that sage can plot standard functions. Sage can also compute integrals symbolically; for example, look carefully at the command which processes $\int \frac{x^2+x+1}{(x-1)^3(x^2+x+2)} \cdot$. This command contains standard LaTeX code to display the integral, but then Sage integrates and returns a typeset copy of the result.

3. ANOTHER SAMPLE

The “Sage and Latexmk” folder contains a more extensive sample file called “example.tex” by Dan Drake. That sample is set up to use pdfflatexmk, but it also works with the standard sage engine by changing the word “pdfflatexmk” to “sage” on the top line. Make a copy of this file in a separate folder and typeset to try out Sage.

4. A DEBUGGING WARNING

After one update to Sage, the sample document included in this folder stopped working. It turned out that the syntax for one Sage command had changed slightly. Breaking just one sage command caused them all to fail. Consequently, if you intend to use Sage together with TeX and suddenly nothing works, a little clever debugging will be required to determine and fix the bad Sage command.

5. FINAL REMARKS

A Sage tutorial is available at the Sage page <https://www.sagemath.org>. It is definitely recommended. Extensive additional documentation is available at the same web page.

% !TEX TS-program = sage

% The following lines are standard LaTeX preamble statements.

```
\documentclass[11pt, onside]{amsart}
\usepackage{geometry}
\geometry{letterpaper}
\usepackage[parfill]{parskip}
\usepackage{graphicx}
\usepackage{amssymb}
\usepackage{epstopdf}
\title{Brief Article}
\author{The Author}
```

% Only one command is required to use Sage within the LaTeX source:

```
\usepackage{sagetex}
```

```
\begin{document}
\maketitle
\section{Introduction}
```

This is an example of using Sage within a \TeX document. We can compute extended values like

$$32^{31} = \text{\sage{32^31}}$$

We can plot functions like $x \sin x$:

```
\sageplot[width=4in]{plot(x * sin( 30 * x), -1, 1)}
```

We can integrate:

$$\int \frac{x^2 + x + 1}{(x - 1)^3 (x^2 + x + 2)} dx = \text{\sage{integrate((x^2 + x + 1) / ((x - 1)^3 * (x^2 + x + 2)), x)}}$$

```
\newpage
```

We can perform matrix calculations:

$$\text{\sage{matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]])^3}}$$
$$AB = \text{\sage{Matrix([[1, 2], [3, 4]])} \sage{Matrix([[5, 6], [6, 8]])} = \sage{Matrix([[1, 2], [3, 4]]) * Matrix([[5, 6], [6, 8]])}}$$

Plots are fun; here is a second one showing $x \ln x$. The `width` command in the source is sent to the `includegraphics` command in LaTeX rather than to Sage.

```
\sageplot[width=5in]{plot(x * ln(x), 0, 2)}
```

Sage understands mathematical constants and writes them symbolically unless it is told to produce a numerical approximation. The term $e \pi$ below is not in the LaTeX source; instead it is the result of a Sage calculation, as is the numerical value on the other side of the equal sign.

The product of e and π is $\text{\sage{pi * e} = \sage{N(pi * e)}}.$

```
\end{document}
```

BRIEF ARTICLE

THE AUTHOR

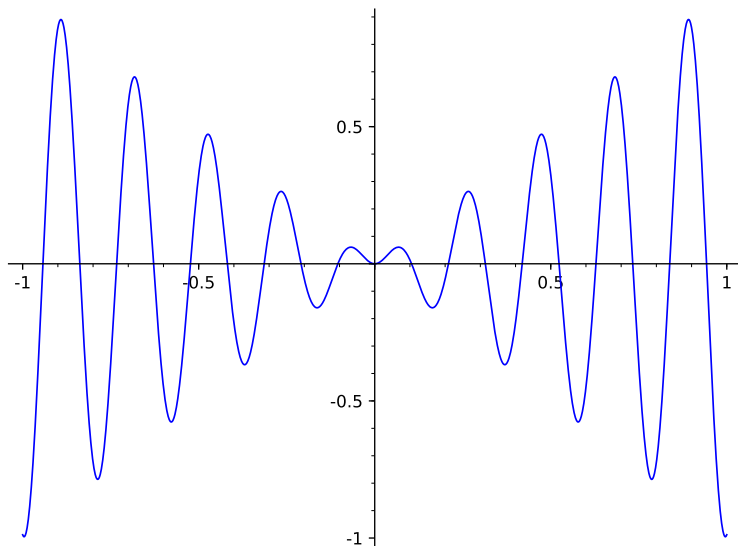
1. INTRODUCTION

This is an example of using Sage within a T_EX document. We can compute extended values like

$$32^{31} = 45671926166590716193865151022383844364247891968$$

$$1324^9 = 12502356138591322345667559424$$

We can plot functions like $x \sin x$:



We can integrate:

$$\int \frac{x^2 + x + 1}{(x-1)^3(x^2+x+2)} dx$$
$$= -\frac{9}{448} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7}(2x+1)\right) - \frac{3(x+1)}{16(x^2-2x+1)} + \frac{5}{128} \log(x^2+x+2) - \frac{5}{64} \log(x-1)$$

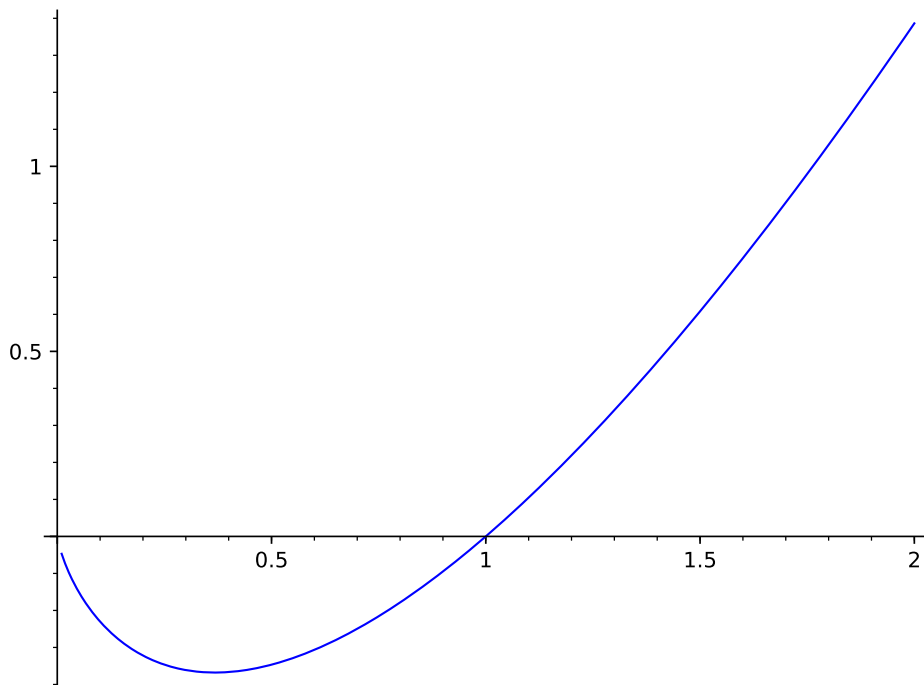
1

We can perform matrix calculations:

$$\begin{pmatrix} 468 & 576 & 684 \\ 1062 & 1305 & 1548 \\ 1656 & 2034 & 2412 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 17 & 22 \\ 39 & 50 \end{pmatrix}$$

Plots are fun; here is a second one showing $x \ln x$. The “width” command in the source is sent to the include graphics command in LaTeX rather than to Sage.



Sage understands mathematical constants and writes them symbolically unless it is told to produce a numerical approximation. The term $e\pi$ below is not in the LaTeX source; instead it is the result of a Sage calculation, as is the numerical value on the other side of the equal sign.

The product of e and π is $\pi e = 8.53973422267357$.