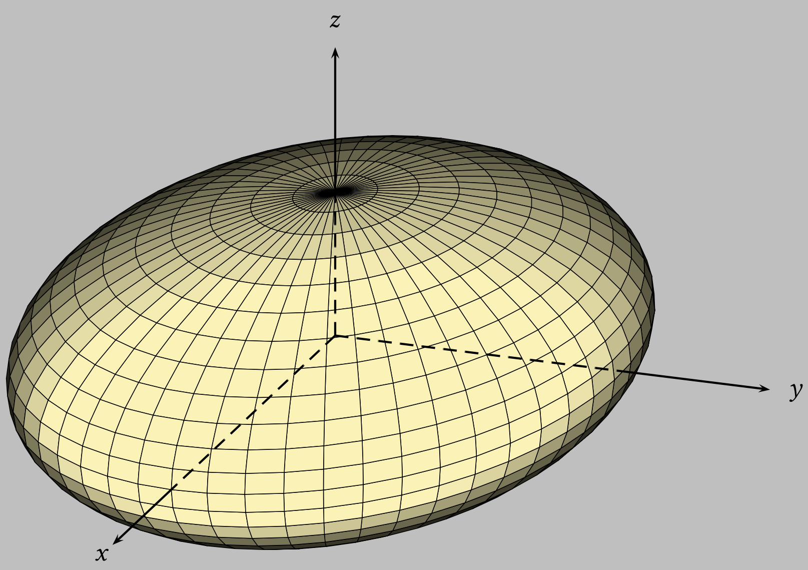
PSTricks

Ellipsoids with PSTricks

June 9, 2026



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Contents

1 Introduction

Code for the title image (exa01)

```
\definecolor{coquille}{rgb}{0.984 0.95 0.718}
\begin{pspicture}(-3,-3)(3,4)
\psset{viewpoint=50 20 20 rtp2xyz,Decran=100,lightsrc=viewpoint}
\psSolid[object=ellipsoid,a=3,b=2,c=1,ngrid=54 24,base=0 360 -90 90,fillcolor=coquille,linewidth=0.01]
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
```

`pst-ellipsoid` is a complementary package for PSTricks for representing ellipsoid solids in 3D. [2] The main package `pst-solides3d` has the new keyword `object=ellipsoid`. This object includes the predefined parameters `a=3`, `b=2`, `c=1`. These values are given as an example; they are the semi-axes of the ellipsoid, they will be automatically placed in the order $a > b > c$ and `base=0 360 -90 90` which allows us to describe the ellipsoid in spherical coordinates $[\theta_1\ \theta_2\ \varphi_1\ \varphi_2]$. [2]

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a\cos\theta\cos\varphi \\ b\sin\theta\cos\varphi \\ c\sin\varphi \end{pmatrix}$$

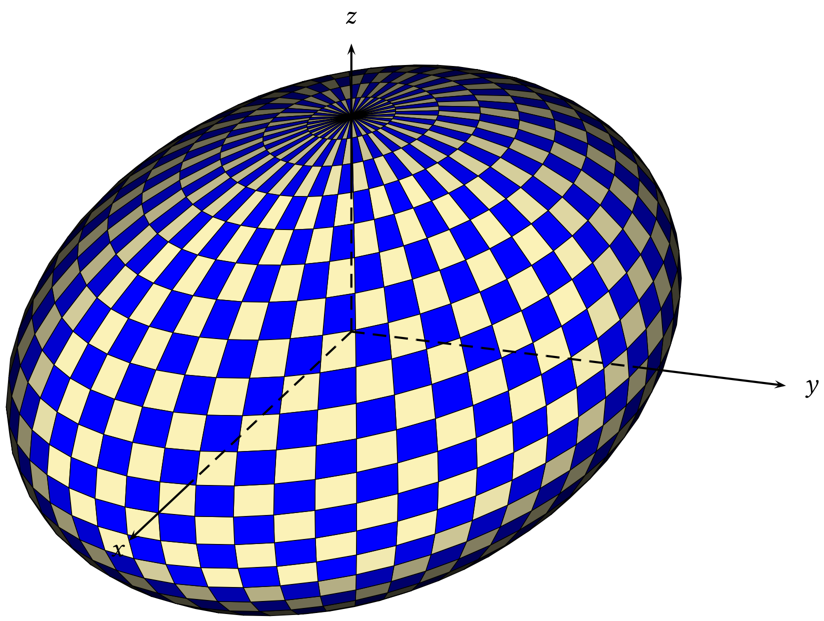
The deafult values are $a1 = 3$, $b1 = 2$, $c1 = 1$. Predifined variables are

$$\begin{aligned}
x_{O1} &= a_1\sqrt{(a_1^2 - b_1^2)/(a_1^2 - c_1^2)} \\
z_{O1} &= c_1\sqrt{(b_1^2 - c_1^2)/(a_1^2 - c_1^2)} \\
y_{O1} &= 0 \\
\mathrm{CosT} &= a_1\frac{\sqrt{(b_1^2 - c_1^2)/(a_1^2 - c_1^2)}}{b_1} \\
\mathrm{SinT} &= c_1\frac{\sqrt{(a_1^2 - b_1^2)/(a_1^2 - c_1^2)}}{b_1} \\
x_P &= 0.95x_{O1} \\
z_P &= x_P z_{O1}/x_{O1} \\
n_X &\stackrel{\text{def}}{=} \mathrm{SinT}\ 0\ \mathrm{CosT} \\
d_P &= |x_P\,\mathrm{SinT} + z_P\,\mathrm{CosT}\,| \\
r_{Cyl} &= b_1\sqrt{1 - (b_1 * d_P/a_1 * c_1)^2}
\end{aligned}$$

These equations can be loaded at the beginning of the code with the macro `\setEllipsoidVars`.

2 Examples

2.1 An ellipsoid with a checkerboard pattern

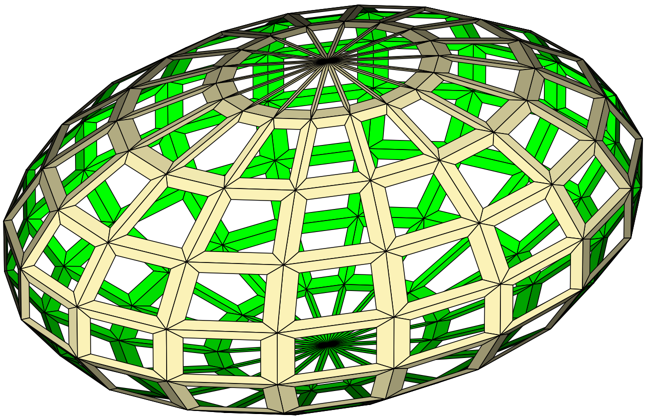


A first example (exa02)

```
\begin{pspicture}(-3,-4)(3,4)
\pstVerb{/n1 54 def /n2 24 def /coquille {0.984 0.95 0.718 setrgbcolor} def}%
\psset{viewpoint=50 20 20 rtp2xyz,Decran=100,lightsrc=viewpoint}
\psSolid[object=ellipsoid,a=3.5,b=2,c=1.5,ngrid=n1 n2,base=0 360 -90 90,fillcolor=blue,linewidth=0.01,
    fcol=0 2 n1 2 sub {/i exch def
             i n2 mul 2 i n2 mul n2 add 1 sub {(coquille)} for} for
             1 2 n1 1 sub {/i exch def
             i n2 mul 1 add 2 i n2 mul n2 add 1 sub {(coquille)} for} for]
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
```

2.2 Ellipsoid skeleton

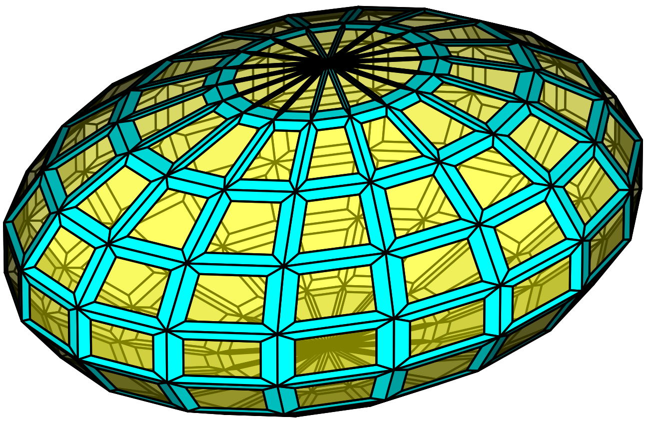
This is a remarkable option developed by Jean-Paul Vignault. If the mesh is very fine (defined by the values of the `ngrid=n1 n2` option), the computation time will be very long! In this case, it is best to save the data for the resulting solid using the `writesolid` option.



Using a data file (exa03)

```
\definecolor{coquille}{rgb}{0.984 0.95 0.718}
\begin{pspicture}(-3,-4)(3,4)
\psset{viewpoint=50 20 20 rtp2xyz,Decran=100,lightsrc=viewpoint}
%\psSolid[object=ellipsoid,a=3,b=2,c=1,ngrid=18 9,base=0 360 -90 90,
%         fillcolor=coquille,linewidth=0.01,
%         affinagecoeff=.7,affinage=all,hollow,
%         action=writesolid,filename=data/ellipsoidaffinage]
\psSolid[object=datfile,filename=data/ellipsoidaffinage,fillcolor=coquille,linewidth=0.01,fcolor=.5 setfillopacity Yellow]
\end{pspicture}
```

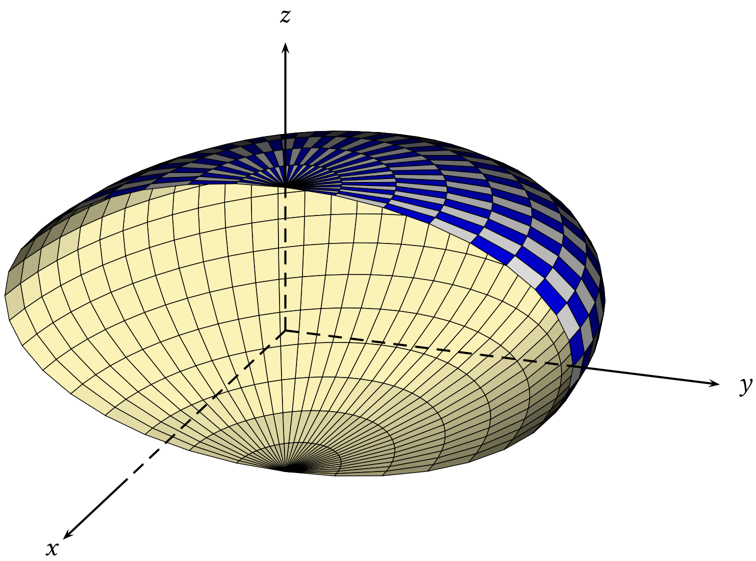
An interesting variant allows you—using the `affinegerm` option—to retain the central face and make it transparent so the interior is visible, or to assign the central faces a color different from that of the framework.



Option `affinegerm` (exa04)

```
\begin{pspicture}(-3,-3)(3,3)
\psset{viewpoint=50 20 20 rtp2xyz,Decran=100,lightsrc=viewpoint}
\psSolid[object=ellipsoid,ngrid=18 9,,a=3,b=2,c=1,base=0 360 -90 90,
         affinagecoeff=.7,affinage=all,fillcolor=cyan,hollow,incolor=yellow!20,
         affinagerm,fcolor=.5 setfillopacity Yellow]
\end{pspicture}
```

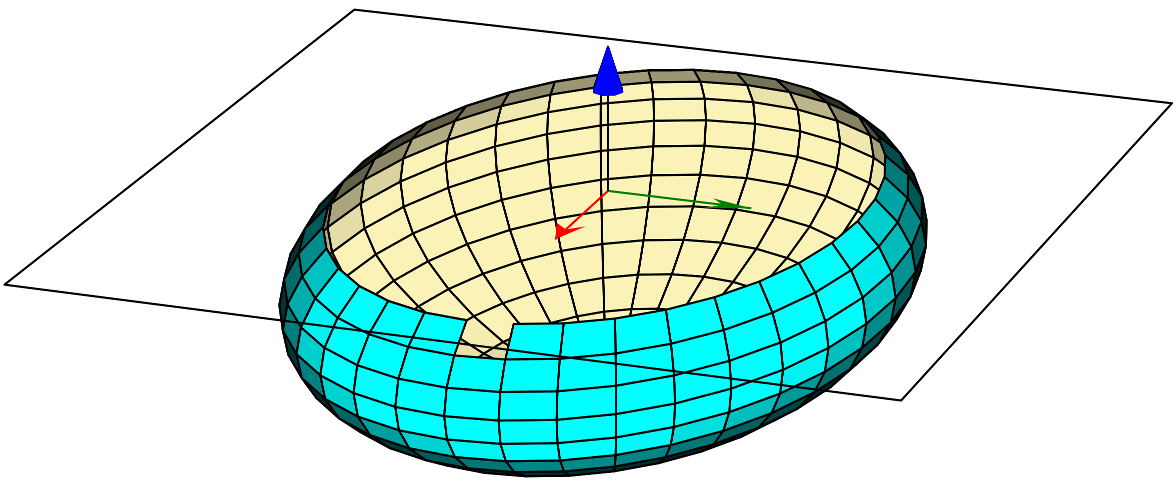
2.3 Semi-ellipsoid



A semi ellipsoid (exa05)

```
\definecolor{coquille}{rgb}{0.984 0.95 0.718}
\begin{pspicture}(-3,-4)(3,4)
\pstVerb{/n1 36 def /n2 18 def  /coquille {0.984 0.95 0.718 setrgbcolor} def}%
\psset{viewpoint=50 20 20 rtp2xyz,Decran=100,lightsrc=viewpoint}
\psSolid[object=ellipsoid,a=3,b=2,c=1,ngrid=n1 n2,base=90 270 -90 90,
  fillcolor=blue,linewidth=0.01,hollow,
% faces extérieures
        fcol=0 2 n1 2 sub {/i exch def
             i n2 mul 2 i n2 mul n2 add 1 sub {(White)} for} for
             1 2 n1 1 sub {/i exch def
             i n2 mul 1 add 2 i n2 mul n2 add 1 sub {(White)} for} for,
             incolor=coquille
]
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
```

2.4 Section by a horizontal plane



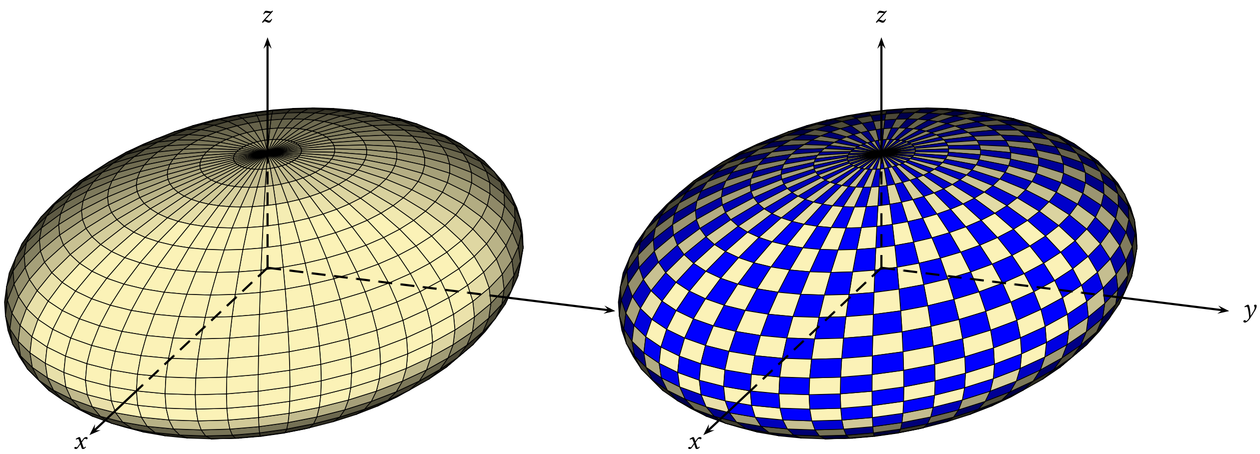
Section by a horizontal plane (exa06)

```
\begin{pspicture}(-3,-4)(3,4)
\pstVerb{/n1 36 def /n2 18 def  /coquille {0.984 0.95 0.718 setrgbcolor} def}%
\psset{viewpoint=50 20 20 rtp2xyz,Decran=100,lightsrc=viewpoint,solidmemory}
\psSolid[object=plan,definition=normalpoint,args={0 0 0.5 [0 0.01 100]},action=none,name=PlanHorizontal]
\psSolid[object=ellipsoid,a=3,b=2,c=1,ngrid=n1 n2,base=0 360 -90 90,
         fillcolor=blue!20,linewidth=0.01,action=none,
         plansepare=PlanHorizontal,name=demiEllipsoid]
\psSolid[object=load,load=demiEllipsoid1,rm=0,hollow,fillcolor=cyan,incolor=coquille]
\psSolid[object=plan,definition=normalpoint,args={0 0 0.5 [0 0.01 100]},action=draw,showBase,base=-3 3 -3 3]
\end{pspicture}
```

3 Slicing an ellipsoid into circular sections

The solution to the problem posed in the 1870 competition for the Montpellier and Aix academies (http://www.numdam.org/item/NAM_1871_2_10_.pdf) was written by Auguste Macé, a student of »Mathématiques spéciales« at the Grenoble lycée. [1] The problem statement consists of three parts; the first is phrased as follows: »Given an ellipsoid, determine the points of contact of the tangent planes parallel to the planes of the circular sections; construct two spheres, each tangent at two of these points (which are symmetric with respect to the major axis): find the equation of the surfaces of revolution of the second degree circumscribed about these two spheres. Classification and discussion of these surfaces.« It is understood that the first step of this part involves determining the umbilical points of the ellipsoid defined by the equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad \text{with } a>b>c$$



Slicing into circular sections (exa07)

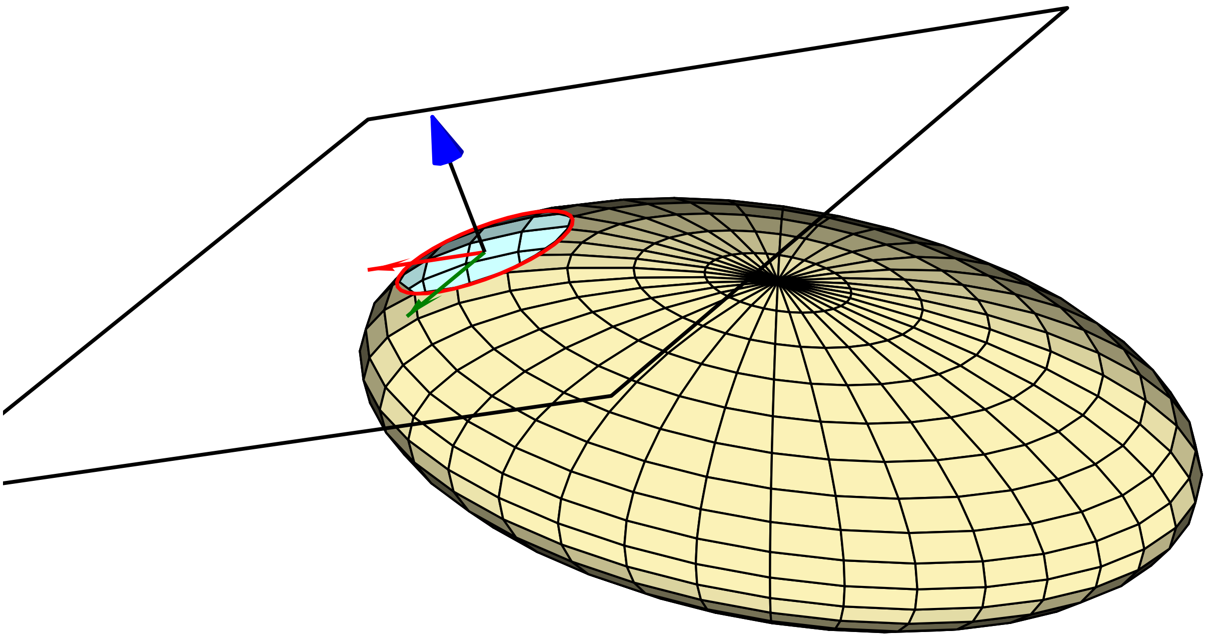
```
\begin{pspicture}(-3,-3)(3,4)
\psset{viewpoint=50 20 20 rtp2xyz,Decran=80,lightsrc=viewpoint,solidmemory}
\psSolid[object=ellipsoid,a=3,b=2,c=1,ngrid=54 24,base=0 360 -90 90,fillcolor=coquille,linewidth=0.01]
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
\hspace{2cm}
\begin{pspicture}(-3,-3)(3,4)
\pstVerb{/n1 54 def /n2 24 def /coquille {0.984 0.95 0.718 setrgbcolor} def}%
\psset{viewpoint=50 20 20 rtp2xyz,Decran=80,lightsrc=viewpoint,solidmemory}
\psSolid[object=ellipsoid,a=3,b=2,c=1,ngrid=n1 n2,base=0 360 -90 90,fillcolor=blue,linewidth=0.01,
       fcol=0 2 n1 2 sub {/i exch def
            i n2 mul 2 i n2 mul n2 add 1 sub {(coquille)} for} for
            1 2 n1 1 sub {/i exch def
            i n2 mul 1 add 2 i n2 mul n2 add 1 sub {(coquille)} for} for]
```

```
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
```

Auguste Macé's demonstration begins with: »we know that the planes of the cyclic sections are parallel to the plane«:

$$x = z\sqrt{\frac{\dfrac{1}{c^2}-\dfrac{1}{b^2}}{\dfrac{1}{b^2}-\dfrac{1}{a^2}}}$$

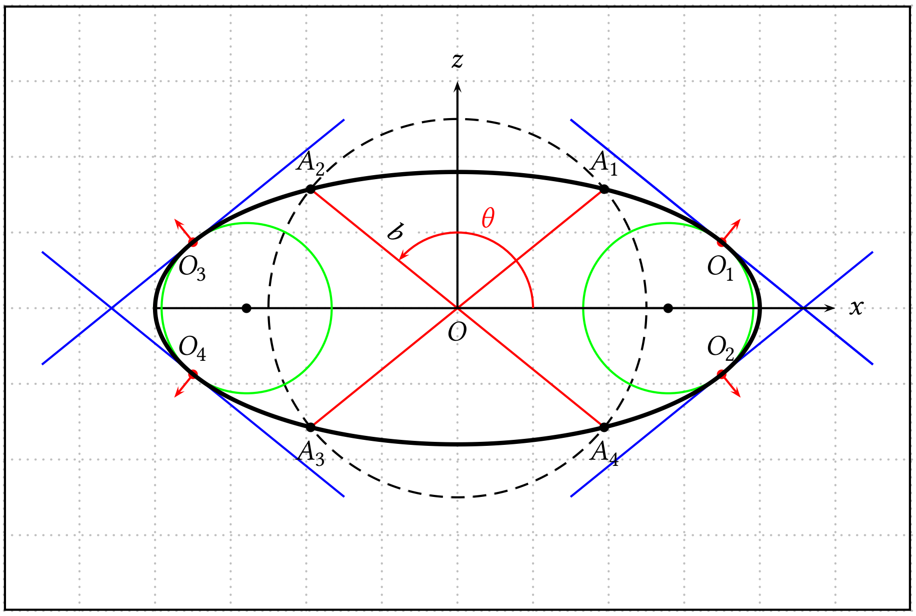
Let us demonstrate that the equation of the circular section plane passing through the origin is indeed written in this form.



Circular sections (exa08)

```
\begin{pspicture}(-5,-5)(5,5)
\psset{viewpoint=100 120 30 rtp2xyz,Decran=200,lightsrc=viewpoint,solidmemory}
\setEllipsoidVars
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=P]
\psSolid[object=ellipsoid,linewidth=0.01,ngrid=36 18,plansepare=P,
        base=0 360 -90 90,name=test,action=none,a=a1,b=b1,c=c1]%
\psSolid[object=load,load=test1,fillcolor=coquille,hollow,rm=0,incolor=cyan!20]%
\psset{plan=P}
\psProjection[object=cercle,args=0 0 radiusCylindre,linecolor=red,linewidth=0.05,range=0 360]
\psSolid[object=plan,definition=normalpoint,action=draw,args={xP 0 zP [nX]},showBase,base=-3 3 -3 3]
\end{pspicture}
```

The circular section passing through the origin is a circle with a radius equal to the semi-axis along the Oy axis: b. To simplify, let us consider the *xOz* plane. The section of the ellipsoid in this plane is an ellipse with semi-major axis a and semi-minor axis c.



The two circular sections passing through O are circles of radius b. Let us determine the coordinates of the points *Ai*.

Ellipse:

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$

Circle of radius b :

$$\begin{cases} x = b\cos\theta \\ z = b\sin\theta \end{cases}$$

The two circular sections passing through O are circles of radius b. Let us determine the coordinates of the points Ai. Ellipse:

$$\frac{b^2\cos^2\theta}{a^2} + \frac{b^2\sin^2\theta}{c^2} = 1 \Rightarrow \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{c^2} = \frac{1}{b^2}$$

We are looking for θ.

$$\frac{\cos^2\theta}{a^2} + \frac{1-\cos^2\theta}{c^2} = \frac{1}{b^2} \Rightarrow \cos^2\theta\left[\frac{1}{a^2} - \frac{1}{c^2}\right] = \frac{1}{b^2} - \frac{1}{c^2}$$

We deduce from this::

$$\cos\theta = \pm\sqrt{\frac{\frac{1}{c^2}-\frac{1}{b^2}}{\frac{1}{c^2}-\frac{1}{a^2}}} \qquad \sin\theta = \pm\sqrt{\frac{\frac{1}{b^2}-\frac{1}{a^2}}{\frac{1}{c^2}-\frac{1}{a^2}}} \qquad \tan\theta = \pm\sqrt{\frac{\frac{1}{b^2}-\frac{1}{a^2}}{\frac{1}{c^2}-\frac{1}{b^2}}}$$

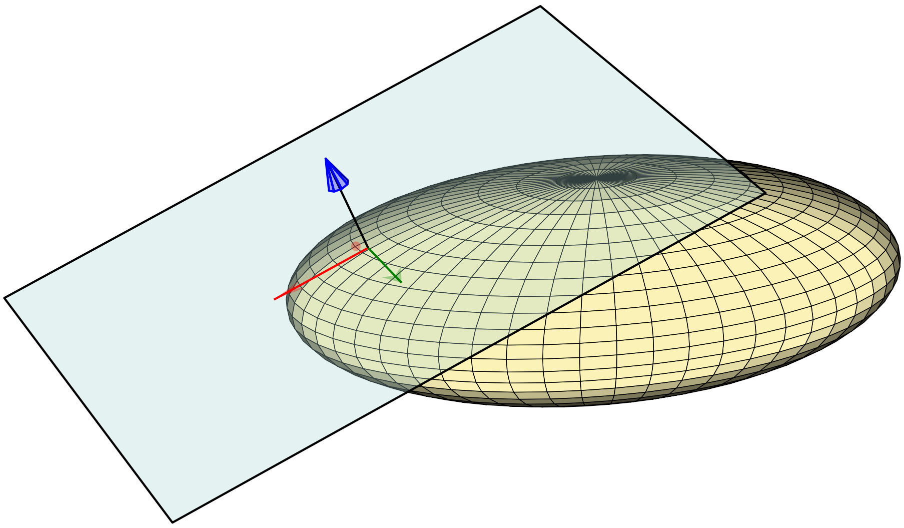
The equations of the planes of circular section passing through O are:

$$z = \pm\sqrt{\frac{\frac{1}{b^2}-\frac{1}{a^2}}{\frac{1}{c^2}-\frac{1}{b^2}}}\,x$$

For the coordinates of the four umbilics, one should refer to Auguste Macé's irrefutable proof. For O1:

$$\begin{cases} x_1 = a\sqrt{\dfrac{a^2-b^2}{a^2-c^2}} \\ y_1 = 0 \\ z_1 = c\sqrt{\dfrac{b^2-c^2}{a^2-c^2}} \end{cases}$$

Here, for example, are the tangent plane and the normal to the ellipsoid at $O1$.



Tangent plane (exa09)

```
\definecolor{coquille}{rgb}{0.984 0.95 0.718}
\begin{pspicture}(-3,-3)(3,5)
\setEllipsoidVars
\psset{viewpoint=50 70 20 rtp2xyz,Decran=70,lightsrc=viewpoint,solidmemory}
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=P1]
\psSolid[object=ellipsoid,a=a1,b=b1,c=c1,ngrid=54 24,base=0 360 -90 90,fillcolor=coquille,linewidth=0.01]
\psset{plan=P1}
\psSolid[object=plan,definition=normalpoint,action=draw**,opacity=0.3,fillcolor=cyan!20,
 args={xP 0 zP [nX]},showBase,base=-3 3 -3 3]
\psPoint(xO1,0,zO1){O1}
\psdot[linecolor=red](O1)
\end{pspicture}
```

The unit normal vector to the cross-sectional planes is:

$$\begin{cases} n_x = \pm\dfrac{c}{b}\sqrt{\dfrac{a^2-b^2}{a^2-c^2}} \\ n_y = 0 \\ n_z = \dfrac{a}{b}\sqrt{\dfrac{b^2-c^2}{a^2-c^2}} \end{cases}$$

If we consider a point on one of the planes located on the ray which joins O to a navel (for example O1(x1, 0, z1)), its coordinates are:

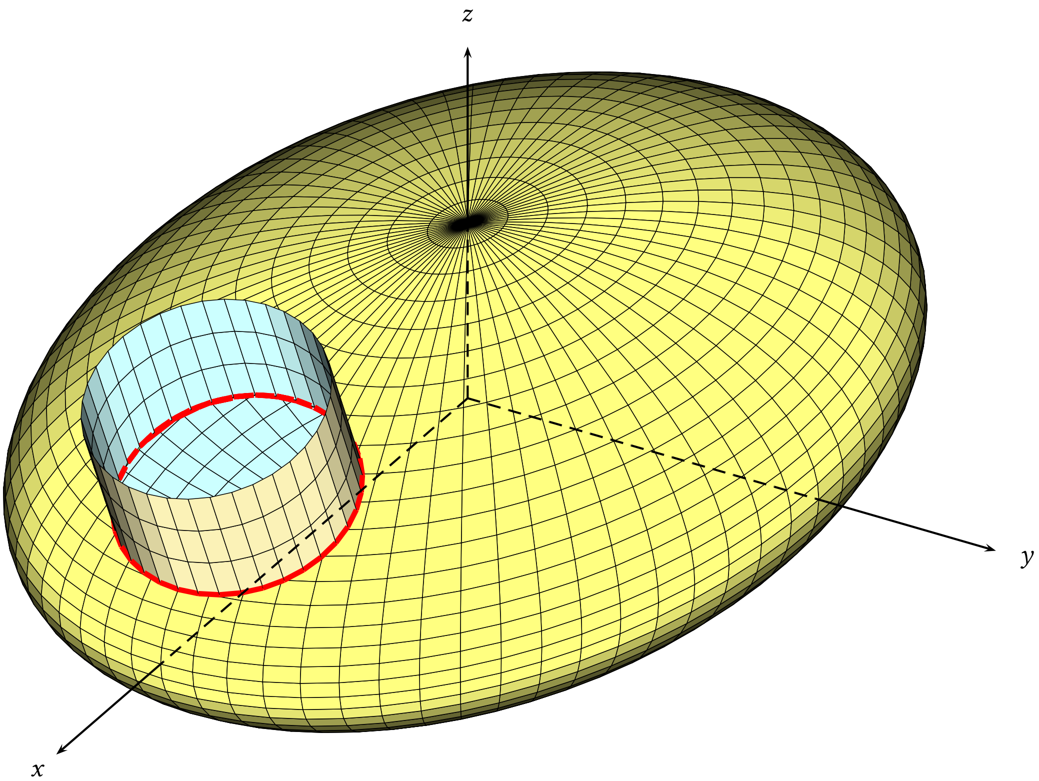
$$\begin{cases} x_P = kx_1 \\ y_P = 0 \\ z_P = \dfrac{x_P z_1}{x_1} \end{cases}$$

The distance from O to the section plane is $d = |x_P n_x + z_p n_z|$, with $0 < d < \dfrac{ac}{b}$, $\dfrac{ac}{b}$ is the distance from O to the umbilical points.

The radius of the circular sections is:

$$r = b\sqrt{1 - \left(\frac{bd}{ac}\right)^2}$$

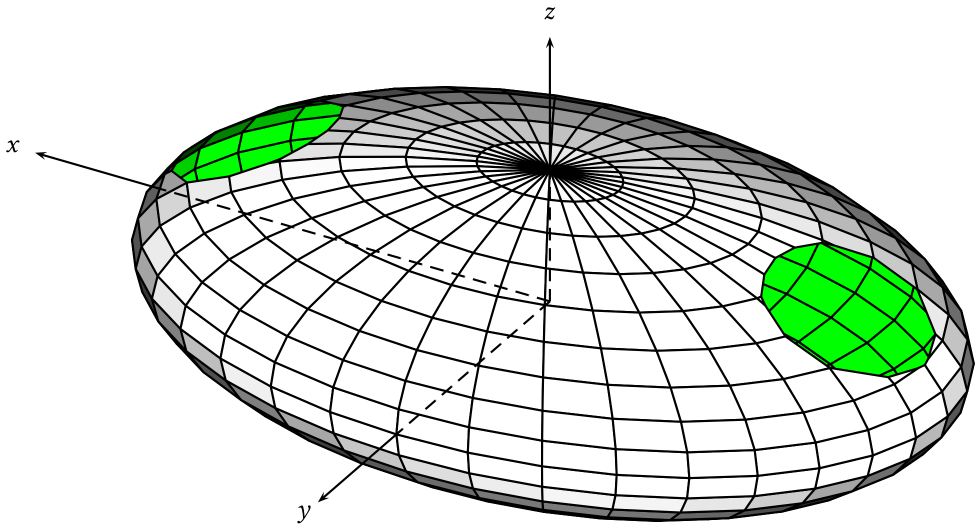
A cylinder of the same radius can be fitted to a circular section of the ellipsoid.



Circular section (exa10)

```
\begin{pspicture}(-7,-5)(7,5)
\setEllipsoidVars
\psset{viewpoint=150 30 30 rtp2xyz,Decran=400,solidmemory,incolor=cyan!20,lightsrc=100 30 30 rtp2xyz}
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=P]
\psSolid[object=ellipsoid,a=a1,b=b1,c=c1,ngrid=72 36,base=0 360 -90 90,
        plansepare=P,name=test,action=none,fillcolor=coquille,linewidth=0.01]
\psSolid[object=load,load=test1,action=none,hollow,rm=0,fillcolor=yellow!50]%
\psSolid[object=cylindre,r=radiusCylindre,h=0.75,RotY=SinT CosT atan,ngrid=3
↪ 36,hollow,fillcolor=coquille,name=cylinder,action=none](xP,0,zP)
\psSolid[object=fusion,base=test1 cylinder,linewidth=0.01,intersectiontype=0,
   intersectionplan=P,intersectionlinewidth=2,intersectioncolor=(rouge)]
\axesIIID(3,2,1)(4,3,2)
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=P]
\end{pspicture}
```

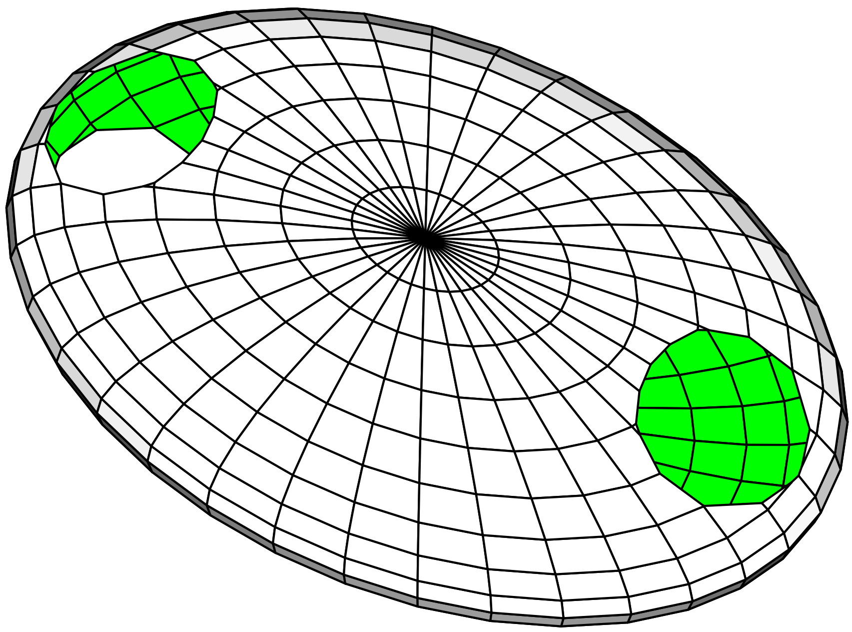
Cutting of two circular sections:



Two sections (exa11)

```
\begin{pspicture}(-7,-5)(7,5)
\setEllipsoidVars
\psset{viewpoint=150 30 30 rtp2xyz,Decran=400,solidmemory,incolor=cyan!20,lightsrc=100 30 30 rtp2xyz}
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=P]
\psSolid[object=ellipsoid,a=a1,b=b1,c=c1,ngrid=72 36,base=0 360 -90 90,
        plansepare=P,name=test,action=none,fillcolor=coquille,linewidth=0.01]
\psSolid[object=load,load=test1,action=none,hollow,rm=0,fillcolor=yellow!50]%
\psSolid[object=cylindre,r=radiusCylindre,h=0.75,RotY=SinT CosT atan,ngrid=3 36,hollow,fillcolor=coquille,name=cylinder,action=none](xP,0,zP)
\psSolid[object=fusion,base=test1 cylinder,linewidth=0.01,intersectiontype=0,
   intersectionplan=P,intersectionlinewidth=2,intersectioncolor=(rouge)]
\axesIIID(3,2,1)(4,3,2)
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=P]
\end{pspicture}
```

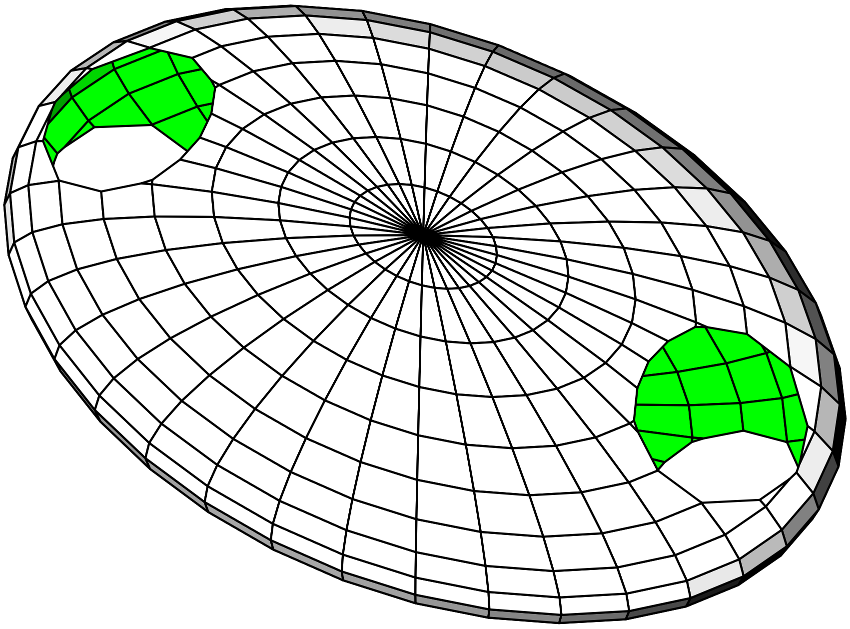
Cutting of three circular sections:



Cutting of three circular sections (exa12)

```
\begin{pspicture}(-5,-4)(5,4)
\psset{viewpoint=100 120 60 rtp2xyz,Decran=200,lightsrc=viewpoint,solidmemory}
\setEllipsoidVars
\psSolid[object=plan,definition=normalpoint,
 args={xP 0 zP [nX]},action=none,name=P]
\psSolid[object=ellipsoid,linewidth=0.01,ngrid=36 18,plansepare=P,
         base=0 360 -90 90,name=test,action=none,a=a1,b=b1,c=c1]%
\psSolid[object=plan,definition=normalpoint, args={xP neg 0 zP [SinT neg 0 CosT]},action=none,name=P2]
\psSolid[object=load,load=test1,fillcolor=yellow,plansepare=P2,name=coupe,action=none]%
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP neg [SinT 0 CosT neg]},action=none,name=P3]
\psSolid[object=load,load=coupe1,plansepare=P3,name=coupeII,action=none]
%\psSolid[object=load,load=coupeII1,action=writesolid,file=3sections]
\psSolid[object=load,load=coupeII1,hollow,rm=0 1 2 ]
\end{pspicture}
```

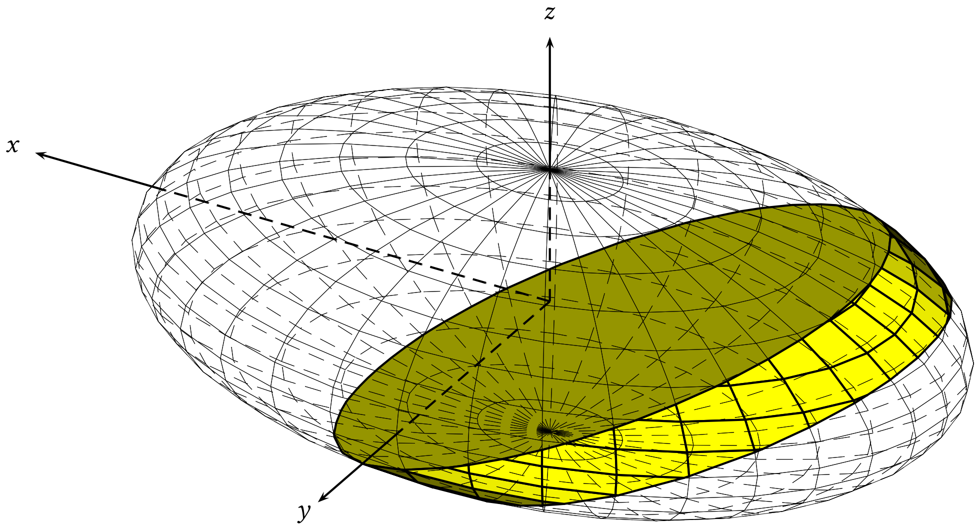
Cutting of four circular sections:



Cutting of four circular sections (exa13)

```
\begin{pspicture}(-5,-4)(5,4)
\psset{viewpoint=100 120 60 rtp2xyz,Decran=200,lightsrc=100 90 60 rtp2xyz,solidmemory}
\setEllipsoidVars
\psSolid[object=plan,definition=normalpoint, args={xP 0 zP [nX]},action=none,name=P]
\psSolid[object=ellipsoid,linewidth=0.01,ngrid=36 18,plansepare=P,
         base=0 360 -90 90,name=test,action=none,a=a1,b=b1,c=c1]%
\psSolid[object=plan,definition=normalpoint, args={xP neg 0 zP [SinT neg 0 CosT]},action=none,name=P2]
\psSolid[object=load,load=test1,fillcolor=yellow,plansepare=P2,name=coupe,action=none]%
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP neg [SinT 0 CosT neg]},action=none,name=P3]
\psSolid[object=load,load=coupe1,plansepare=P3,name=coupeII,action=none]
\psSolid[object=plan,definition=normalpoint,args={xP neg 0 zP neg [SinT neg 0 CosT neg]},action=none,name=P4]
\psSolid[object=load,load=coupeII1,plansepare=P4,name=coupeIII,action=none]
%\psSolid[object=load,load=coupeIII1,action=writesolid,file=4sections]
\psSolid[object=load,load=coupeIII1,hollow,rm=0 1 2 3 ]
\end{pspicture}
```

A slice with a circular cross-section (a disk) is cut:

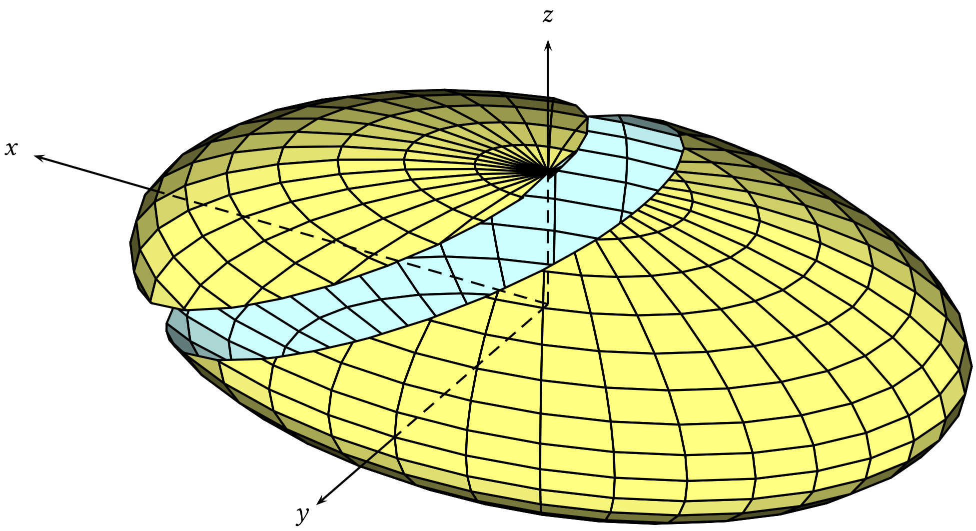


A circular cross-section (exa14)

```
\definecolor{coquille}{rgb}{0.984 0.95 0.718}
\begin{pspicture}(-3.5,-4)(3.5,4)
\psset{viewpoint=100 120 30 rtp2xyz,Decran=200,lightsrc=viewpoint,solidmemory,a=3,b=2,c=1}
\codejps{
  /ngrid  [36 18] def
  /base [0 360 -90 90] def
  ellipsoid
  /xO1 a a dup mul b dup mul sub  a dup mul c dup mul sub div sqrt mul def
  /zO1 c b dup mul c dup mul sub  a dup mul c dup mul sub div sqrt mul def
  /yO1 0 def
  % distance de O à O1
  /dO xO1 dup mul zO1 dup mul add sqrt def
  /CosT b dup mul c dup mul sub a dup mul c dup mul sub div sqrt b div a mul def
  /SinT a dup mul b dup mul sub a dup mul c dup mul sub div sqrt b div c mul def
  % distance de O à O1
  % les coordonnées d'un point du plan de coupe
  /xP 0.5 xO1 mul def
  /zP xP zO1 mul xO1 div def
  % la normale au plan de coupe circulaire
  /nX {SinT 0 CosT} def
  % distance au plan de coupe
  /dP xP SinT mul zP CosT mul add abs def
  /radiusCylindre b 1 b dP mul a c mul div dup mul sub sqrt mul def
[ nX dP ]
solidplansepare
/ellipsoide2 exch def
/ellipsoide1 exch def
ellipsoide1
/xP 0.2 xO1 mul def
/zP xP zO1 mul xO1 div def
/dP xP SinT mul zP CosT mul add abs def
[ nX dP ]
solidplansepare
/ellipsoide2 exch def
/ellipsoide1 exch def
ellipsoide2
dup (yellow) outputcolors
%dup [0.1 0.2] solidputhuecolors
solidlightOn
drawsolid**
0.1 setlinewidth
ellipsoid
```

```
drawsolid
}
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
```

Remove one washer and keep the two remaining parts.



Remaining two parts (exa15)

```
\begin{pspicture}(-5,-5)(5,5)
\psset{viewpoint=100 120 30 rtp2xyz,Decran=200,lightsrc=viewpoint,solidmemory,incolor=cyan!20,fillcolor=yellow!50}
\setEllipsoidVars
\pstVerb{/xP 0.6 x01 mul def
/zP xP z01 mul x01 div def
/dP {2 dict begin /yP exch def /xP exch def xP SinT mul zP CosT mul add abs end} def
/radiusCylindre {2 dict begin /yP exch def /xP exch def b1 1 b1 xP yP dP mul a1 c1 mul div dup mul sub sqrt mul} def
}
\psSolid[object=plan,definition=normalpoint,args={xP 0 zP [nX]},action=none,name=Plan1]
\psSolid[object=ellipsoid,linewidth=0.01,ngrid=36 18,plansepare=Plan1,
         base=0 360 -90 90,name=coupeA,action=none,a=a1,b=b1,c=c1]%
\psSolid[object=load,load=coupeA0,action=none,name=part0,rm=0,hollow]%
% les coordonnées d'un point du deuxième plan de coupe
\pstVerb{/xP2 0.4 x01 mul def /zP2 xP2 z01 mul x01 div def}%
\psSolid[object=plan,definition=normalpoint,args={xP2 0 zP2 [SinT 0 CosT]},action=none,name=Plan2]
\psSolid[object=ellipsoid,linewidth=0.01,ngrid=36 18,plansepare=Plan2,
         base=0 360 -90 90,name=coupeB,action=none,a=a1,b=b1,c=c1]%
\psSolid[object=load,load=coupeB1,action=none,name=part1,rm=0,hollow]%
\psSolid[object=fusion,base=part0 part1]
\axesIIID(3,2,1)(4,3,2)
\end{pspicture}
```

References

[1] Auguste Macé. "Solution de la question proposée au concours pour les deux académies de Montpellier et d'Aix (année 1870)". In: *Nouvelles annales de mathématiques*. 2nd ser. 10 (1871), pp. 17–26. URL: http://www.numdam.org/item?id=NAM_1871_2_10__17_1%3E (visited on 06/07/2026).

[2] Wikipedia. *Ellipsoid*. URL: https://en.wikipedia.org/wiki/Ellipsoid (visited on 06/09/2026).

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